Review of the Literature: Maintenance and Rehabilitation Costs for Roads (Risk-based Analysis)

By:
Noppadol Piyatrapoomi and Arun Kumar
RMIT University, Melbourne, Australia

In Collaboration with

Neil Robertson and Justin Weligamage
Queensland Department of Main Roads, Brisbane, Australia

Project Partners
RMIT University
Queensland Department of Main Roads
Queensland Department of Public Works
Queensland University of Technology

Project: 2003-029-C
Maintenance Cost Prediction for Roads

Research Program: C
Delivery and Management of Built Assets

Report: 2003-029-C/001

September 2004

*ALL RIGHTS RESERVED. NO PART OF THIS REPORT MAY BE REPRODUCED WITHOUT PRIOR PERMISSION
TABLE OF CONTENT

Preface ii
Executive Summary iii

1. Introduction 1

2. Review of the Literature 1
   2.1 Risk-based life-cycle costing of infrastructure Rehabilitation and construction alternatives 2
   2.2 Optimum Flexibility Pavement Life-Cycle Analysis Model 3
   2.3 Highway Development Decision-Making under Uncertainty: A Real Option Approach 6
   2.4 Two Probabilistic Life-Cycle Maintenance Models for Deteriorating Civil Infrastructures 7
   2.5 Risk-Based Expenditure Allocation for infrastructure Improvement 9

3. Proposed Methodology for Assessing Risk of Errors in Budget Estimates for Road Maintenance and Rehabilitation 10

4. Conclusions 13

List of References 14

Author Biographies 15
PREFACE

This report presents a review of the literature on risk assessment of errors in budget estimates for road maintenance and rehabilitation. Risk of errors in budget estimates arises from uncertainties and variability in input parameters. Uncertainties and variability of input parameters arise from the randomness of events such as climatic conditions, soil conditions and road user traffic. This report presents how current practices incorporate uncertainty and variability of road asset conditions and other critical input parameters in assessing risk of errors in budget estimates for road maintenance and rehabilitation.

The authors wish to acknowledge the Cooperative Research Centre for Construction Innovation (CRC CI) for their financial support. The authors also wish to thank Mr. Neil Robertson, Mr. Justin Weligamage, and Mr. John Spathonis of Queensland Department of Main Roads, and Mr. Dale Gilbert of Queensland Department of Public Works for their support.
EXECUTIVE SUMMARY

Errors in budget estimates for infrastructure asset maintenance and rehabilitation and life-cycle cost estimates have been recognised as an important issue. The degree of errors depends on the variability and uncertainties in infrastructure asset conditions and other critical parameters that relate to deterioration rates of infrastructures. Information on the degree of errors in budget estimates could assist decision-makers to make decisions on budget allocation with greater confidence.

Researchers have applied statistical methods and probability-based risk and reliability methods to assess the degree of errors in budget estimates. However, there is limited information on assessing risk of errors in budget estimates for infrastructure asset management especially for road infrastructure asset management. The aim of this review of the literature is to provide an update on current practices in this area, an outcome of the first stage of a CRC project on “Investment Decision-Framework for Infrastructure Asst Management”. For the second stage of project, the update will provide current information which can be drawn upon to formulate a methodology for assessing risk of errors in life-cycle budget/cost estimates for road maintenance and rehabilitation whilst taking into account overall variability of critical input variables in the analysis.

A methodology for assessing risk of errors in budget estimates has been proposed in this report. This method is based on a simulation technique called Latin hypercube sampling technique that can simulate a small number of data points to represent the overall variability of critical input parameters. This simulation technique can substantially reduce input data preparation to a practical level.

In this method, road networks are divided into small sections. The variability of road asset condition in each section is quantified. Road deterioration prediction models and the variability of road asset condition are used to predict the variability of road condition for each road section. The errors in deterioration prediction models can be incorporated in the analysis. The budget estimates for road maintenance and rehabilitation are estimated from the variability of future road condition variability. The selection of appropriate road works and budgets are obtained from road work standard and optimisation techniques.

The result of the analysis will be a probability distribution of a life-cycle budget estimate in which the overall critical variability of input variables has been incorporated in the analysis. Budget analysts will be able to refine the accuracy of budget estimates using the output probability distribution of budget estimates. For instance, a budget could be selected for a 90% or 95% confidence that it will not be exceeded. Or a lower budget could be selected with at a higher risk level of being exceeded. Budget analysts will be aware of the degree of risk of errors when a budget is selected.
1. Introduction

Realistic estimates of short- and long-term (strategic) budgets for maintenance and rehabilitation of road assessment management should consider the stochastic characteristics of asset conditions of the road networks so that the overall variability of road asset data conditions is taken into account.

The probability theory has been used for assessing life-cycle costs for bridge infrastructures by Kong and Frangopol (2003), Zayed et.al. (2002), Kong and Frangopol (2003), Liu and Frangopol (2004), Noortwijk and Frangopol (2004), Novick (1993). Salem 2003 cited the importance of the collection and analysis of existing data on total costs for all life-cycle phases of existing infrastructure, including bridges, road etc., and the use of realistic methods for calculating the probable useful life of these infrastructures (Salem et. al. 2003). Zayed et. al. (2002) reported conflicting results in life-cycle cost analysis using deterministic and stochastic methods. Frangopol et. al. 2001 suggested that additional research was required to develop better life-cycle models and tools to quantify risks, and benefits associated with infrastructures.

It is evident from the review of the literature that there is very limited information on the methodology that uses the stochastic characteristics of asset condition data for assessing budgets/costs for road maintenance and rehabilitation (Abaza 2002, Salem et. al. 2003, Zhao, et. al. 2004). Due to this limited information in the research literature, this report will describe and summarise the methodologies presented by each publication and also suggest a methodology for the current research project funded under the Cooperative Research Centre for Construction Innovation CRC CI project no 2003-029-C.

2. Review of the Literature

There are two types of asset management for road infrastructures: preventive maintenance and essential maintenance or rehabilitation for life cycle of infrastructures. If preventive maintenance is not conducted, it will become more costly to maintain the infrastructure in good service and safe condition at a later stage. Essential maintenance or rehabilitation is required to make the infrastructure safe for users.

The development of the systematic life cycle cost analysis and deterministic assumption of asset performance condition curves enables road asset management engineers and managers to prepare preventive maintenance and essential rehabilitation planning more effectively than conventional methods of pre-fund allocation or the perceived urgency of maintenance/rehabilitation. Most modelling approaches used optimising benefit/cost analysis techniques for finding optimal resource allocations. However, the deterministic assumption of the performance condition is not a valid assumption due to apparent variability and uncertainty observed in asset condition data. Some researchers have introduced methods to include reliability concepts in the process of determining life-cycle costs (Salem et. al. 2003, Zho, et. al. 2004, Kong & Frangopol 2003, Noortwijk & Frangopol 2004). These methods are reviewed in the following sections.
2.1 Risk-based life-cycle costing of infrastructure rehabilitation and construction alternatives (Salem et. al. 2003)

This research presented a new approach for estimating life-cycle costs and evaluating infrastructure rehabilitation and construction alternatives derived from probability theory and simulation application. Salem et. al. (2003) argued that most approaches in assessing life-cycle costs for civil infrastructure construction and rehabilitation alternatives assumed a deterministic behaviour for the service life of an infrastructure, which is not a valid assumption. The risk-based life cycle cost model developed by Salem et al. (2003) considers the time to failure of each pavement rehabilitation/construction alternative and provides additional knowledge about the uncertainty levels that accompany the estimated life cycle costs. In this study, highway pavement data were used to demonstrate the model concept and development. The element of uncertainty is introduced through the parameters of the probability distributions fitted to infrastructure time-to-failure data. These parameters are incorporated into the model using random sampling of random variables from the fitted distributions. Salem et. al. 2003 introduced the risk-based life-cycle costing model as given below;

\[
LCC_{a,n} = C_{i_a} + \sum_{t=0}^{n} pwf_{r,t}(C_f)_{a,t} + \sum_{t=0}^{n} pwf_{r,t}(C_m)_{a,t} + \sum_{t=0}^{n} pwf_{r,t}(C_u)_{a,t} - pwf_{r,n}(S_{v})_{a,n}
\]

\[
\text{.................(1)}
\]

Where \(LCC_{a,n}\) = present worth of life-cycle cost for alternative \(a\), for analysis period of \(n\) years; \(C_{i_a}\) = initial capital cost for construction for alternative \(a\); \(pwf_{r,t}\) = present worth factor of future amount at time \(t\) years at discount rate of \(r\); \(C_{f,a,t}\) = cost of failure (rehabilitation) for alternative \(a\) at year \(t\); \(C_{m,a,t}\) = cost of maintenance for alternative \(a\) at year \(t\); \(C_{u,a,t}\) = user costs due to failure for alternative \(a\) at year \(t\); \(pwf_{r,n}\) = present worth factor of future amount at end of analysis period at discount rate of \(r\); \(S_{v,a,n}\) = salvage value for alternative \(a\) at the end of analysis period \(n\).

The variability of \(C_{i_a}, C_{f,a,t}, C_{m,a,t}, C_{u,a,t}\) and \(S_{v,a,n}\) is quantified by probability distributions.

The pavement performance indicator is rated by the riding comfort index (RCI) on a scale of 0 to 10. The time to failure, when rehabilitation is needed, is considered when the RCI value drops below 5.5. The times to failure were calculated from historical data for different pavement types. The statistical distribution and the distribution parameters of the time to failure for each pavement type were established. These parameters were then used as input data for simulating times to failure, which were then used to predict the probability distributions of life-cycle costs of construction/rehabilitation alternatives.

Salem et. al. 2003 applied their risk-based life-cycle costing model to investigate life-cycle costs for three types of pavement alternatives for a rehabilitation project for a 22.33 km highway in central Alberta east of Edmonton in Canada.

The three alternatives were identified as:

1. Alternative A (SC): Soil-cement pavement initially costing $238,000/km in construction
2. Alternative B (FD): Full-depth pavement initially costing $250,000/km in construction
3. Alternative C (GB); Granular-based pavement initially costing $260,000/km in construction

Each alternative will require a non-structural overlay for their first rehabilitation need, which will cost $57,000/km.

The statistical parameters and distributions of the service life for the three pavement types and rehabilitation alternatives are quantified and given in Table 1.

Table 1 Statistical parameters and distributions of the service life for the three pavement types and rehabilitation alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Distribution Construction</th>
<th>Distribution Parameters</th>
<th>Distribution Rehabilitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (SC)</td>
<td>Weibull</td>
<td>3.628&lt;sup&gt;a&lt;/sup&gt;</td>
<td>14.66&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.635&lt;sup&gt;b&lt;/sup&gt;</td>
<td>10.24&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>B (FD)</td>
<td>Weibull</td>
<td>4.331&lt;sup&gt;a&lt;/sup&gt;</td>
<td>16.58&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.995&lt;sup&gt;b&lt;/sup&gt;</td>
<td>9.509&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>C (GB)</td>
<td>Lognormal</td>
<td>16.04&lt;sup&gt;a,c&lt;/sup&gt;</td>
<td>5.94&lt;sup&gt;a,d&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.315&lt;sup&gt;b&lt;/sup&gt;</td>
<td>12.22&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> For distribution (construction)
<sup>b</sup> For distribution (rehabilitation)
<sup>c</sup> Mean
<sup>d</sup> Standard deviation

Salem et. al. 2003 used the initial construction and rehabilitation costs of the three alternatives, the probability distributions and their parameters given in Table 1 and the risk-based life cycle model to investigate the probability distributions of the three alternative life cycle-costs.

In this risk-based life-cycle cost model, the statistical distributions and the statistical parameters derived from the time to failure for each type of pavement were assigned as the statistical distributions and the statistical parameters of the initial construction and rehabilitation costs for each type of pavement.

Salem et. al. 2003 assumed the available budget to carry out the work on the highway was $6,243,700 or $290,000 per kilometre for the analysed highway section. Using 8% as the discount rate, the result indicated that for alternative A there is a 77.5% chance that the total life-cycle cost will be within the budget. For alternative B, there is an 82.5% chance that the total life-cycle cost will be within the budget. For alternative C, there is a 70% chance that the total life-cycle cost will be within the budget. According to the probabilities, alternative B is the one that the decision maker should select, since it provides the best chance for meeting the budget.

2.2 Optimum Flexibility Pavement Life-Cycle Analysis Model (Abaza 2002)

Abaza 2002 suggested a life cycle cost assessment model that yields an optimum maintenance and rehabilitation plan. The model incorporated into the optimisation process both performance and cost associated with a life-cycle analysis period for a
given pavement performance. A single life-cycle indicator called “life-cycle disutility” was introduced and defined as the ratio of cost to performance.

**Optimum Pavement Life-Cycle Performance**

The optimum life cycle performance is based on relative performance, mathematically given in Equation 2. The relative performance is defined as the ratio of the area corresponding to a pavement life cycle curve to that of a perfect performance.

\[
RP_{LC} = \frac{A_{LC}}{(P_o - P_f)T_{m+1}}
\]

Where \( RP_{LC} \) = pavement life-cycle relative performance; \( A_{LC} \) = area under pavement life-cycle curve; \( P_o \) = initial performance condition index value of original pavement; \( P_f \) = pavement life-cycle failure performance index; \( T_{m+1} \) = length of a life-cycle analysis in years.

Abaza 2002 used AASHTO’s formula (AASHTO 1993) in constructing pavement life-cycle performance curves. The AASHTO’s formula is given as follows;

\[
\log(W_{80}) = Z_R S_o + 9.36 \log(SN + 1) + \frac{\log(\Delta PSI)}{9.2 - 1.5} + \frac{1094}{(SN + 1)^{5.19}}
\]

Where \( W_{80} \) = number of 80 kN equivalent single axle load applications estimated for a selected design period and design lane; \( Z_R \) = standard normal deviated for a specified reliability level; \( S_o \) = combined standard error of the traffic prediction and performance prediction; \( \Delta PSI \) = difference between the initial or present serviceability index \( P_o \) and the terminal serviceability index \( P_t \); \( SN \) = design structural number indicative of the total required pavement thickness; and \( M_R \) = sub-grade resilient modulus.

Abaza 2002 approach in constructing the pavement performance curve was to calculate the incremental 80 kN equivalent single axle load by specifying varying values of the incremental change in the present serviceability index (\( \Delta PSI \)).

**Optimum Life-Cycle Cost**

The costs incurred over the life-cycle analysis period include the construction cost of original pavement, rehabilitation cost of a number of major rehabilitation cycles, routine maintenance cost, and user cost. The present worth of the pavement life-cycle cost for a fixed life-cycle is calculated as follows;

\[
P_{LC} = C_c + M_c \times f(P/A,r,T_{m+1}) + \sum_{j=1}^{m} R_j \times f(P/F,r,T_j)
\]
\[ f\left(\frac{P}{A}, r, T_{m+1}\right) = \frac{(1+r)^{T_{m+1}} - 1}{r(1+r)^{T_{m+1}}} \]
\[ f\left(\frac{P}{F}, r, T_j\right) = \frac{1}{(1+r)^j} \]

Where \( P_{LC} \) = pavement life-cycle present worth cost for a given maintenance and rehabilitation; \( C_c \) = initial construction cost of original pavement; \( M_c \) = annual routine maintenance and added user costs; \( R_j \) = future rehabilitation cost of the \( j^{th} \) cycle \((j=1, 2, \ldots, m)\); \( T_{m+1} \) = length of life-cycle analysis period in years, \( r \) is annual interest rate; \( m \) = number of deployed major rehabilitation cycle in an analysis period; \( T_j \) = scheduled rehabilitation time of the \( j^{th} \) cycle in years; \( f(P/A, r, T_{m+1}) \) = factor converting a uniform annual cost to a present cost; \( f(P/F, r, T_j) \) = factor converting future cost to a present one.

For a variable life-cycle cost analysis, the present worth cost can be calculated as follows;

\[ EA_{LC} = P_{LC} \times f\left(\frac{A}{P}, r, T_{m+1}\right) \tag{5} \]
\[ f\left(\frac{A}{P}, r, T_{m+1}\right) = \frac{r(1+r)^{T_{m+1}}}{(1+r)^{T_{m+1}} - 1} \]

Where \( EA_{LC} \) = pavement life-cycle equivalent annual cost; \( P_{LC} \) = pavement life-cycle present worth cost obtained from Eq. (4) for variable \( T_{m+1} \) periods; and \( f(A/P, r, T_{m+1}) \) = factor converting a present cost to an equivalent uniform annual one.

**Optimum Pavement Life-Cycle Cost Disutility**

The life-cycle disutility parameter was used by Abaza 2002 as a means for an effective single indicator. The pavement life-cycle disutility is defined as the ratio of life-cycle cost to life-cycle performance represented by the area under the life-cycle performance curve. The life-cycle disutility parameter is given as follows for a fixed analysis period;

\[ U_{LC} = \frac{P_{LC}}{A_{LC}} \tag{6} \]

Where \( U_{LC} \) = pavement life-cycle disutility; \( P_{LC} \) = pavement life-cycle present worth cost; \( A_{LC} \) = area under the performance curve.

The life-cycle disutility parameter for the varying period is given as follows;

\[ U_{LC} = \frac{EA_{LC}}{A_{LC}/T_{m+1}} \tag{7} \]

Where \( U_{LC} \) = pavement life-cycle disutility; \( EA_{LC} \) = pavement life-cycle equivalent annual cost; \( A_{LC} \) = area under a life-cycle performance curve; \( T_{m+1} \) = pavement life-cycle analysis period (years).
An optimum maintenance and rehabilitation project is identified by the minimum life-cycle disutility value ($U_{LC}$) and the maximum value of relative performance value ($RPLC$).

### 2.3 Highway Development Decision-Making under Uncertainty: A Real Options Approach (Zhao, et. al. 2004)

Zhao, et. al. 2004 presented a multistage stochastic model for decision-making in highway development, operation, expansion and rehabilitation using real option approach. This model accounted for three uncertainties; namely traffic demand, land price and highway deterioration as well as their interdependence. They argued that the demand and revenue projection for the life-cycle of a highway are embedded with multiple uncertainties due to changes in political, societal and environmental contexts. The optimality of decision-making was conventionally limited to predetermined policies or plans. Uncertainties such as demand, costs, revenues and service quality were often interrelated and could not be dealt with in isolation. They adopted the concept of financial real option theory. The flexibility in real option may turn uncertainties into opportunities. The proposed model and solution algorithm produced decision-making optimality that was generally not well defined in traditional policy-based approaches for highway development, operation, expansion and rehabilitation.

\[
F_t(v_t;X_t) = f_t(v_t;X_t) + \max_{u_t} \left\{ e^{-r} E_t \left[ F_{t+1}(v_{t+1};X_{t+1}) \right] - c_t(u_t, v_t) \right\}
\]

\[ v_t = (n_t, w_t) \]
\[ X_t = (Q_t, P_t, I_t) \]
\[ u_t = (\Delta n_t, \Delta w_t, h_t) \]

Where $F_t(v_t;X_t) =$ the value indicating the total value (expected profit) of the system for the remaining period at state ($v_t$) at time $t$; $v_t =$ a vector (collection) of state of variables indicating the number of lanes ($n_t$) and the right-of-width ($w_t$) at time $t$; $X_t =$ a factor (collection) of the underlying uncertainties of traffic demand ($Q_t$), land price ($P_t$) and pavement condition index ($I_t$) at time $t$; $u_t =$ a vector (collection) of decision variables indicating the number of lane to be expanded ($\Delta n_t$), the width of the right-of-way ($\Delta w_t$) and rehabilitation ($h_t$) at time $t$; $c_t(u_t, v_t) =$ cost incurred by making decision $u_t$ under state $v_t$ at time $t$; $f(v_t;X_t) =$ the system revenue at time $t$ or current system revenue; $t =$ index for time ($t = 0, ..., T$) in years; $T =$ length of the planning horizon over the life cycle of the highway system; $E_t =$ expectation operator and the subscript $t =$ expectation based on the available information for the uncertainty $X_t$; $r =$ risk-adjusted discount rate over one year.

The model supports the decision-making in the right-of-way acquisition, highway expansion and rehabilitation under uncertainties by utilising flexibility in making decision. For instance, assuming that a state $v_t$ at time $t$, the uncertainty vector $X_t$ is revealed. Upon observing $X_t$, the decision-maker (i) must realise the current system of the revenue, $f(v_t;X_t)$, and (ii) can strategically utilise the available flexibility by making decisions $u_t$ with a cost of $c_t(u_t, v_t)$ incurred.
The model compares the initial condition of the revenue system, \( F_0(v_0, X_0) \) and the optimal value representing the maximum expected system value obtained from the last step of the recursive relation in equation (8).

**Modelling the revenue function**

The highway capacity is assumed to be linear function of the number of lanes. For a highway to be constructed and or under construction, the revenue is assumed to be zero. For an existing highway, the revenue comes from two sources; traffic flow and land use. Thus, the revenue function is given as follows;

\[
 f_t(v_t; X_t) = \text{revenue from traffic flow} + \text{revenue from land}
\]

**Modelling the cost function**

The cost function is assumed to be a linear function, which is the summation of expansion cost, land acquisition cost for right-of-way and rehabilitation cost.

\[
 c_t(u_t, v_t) = \text{expansion costs} + \text{acquisition cost for right-of-way} + \text{cost for rehabilitation}
\]

**Solution Algorithm**

If the expected system value is available at time \( t \) under \((u_t, v_t)\), one could know the expected system profit for the next time period when the uncertainty \( X_t \) is revealed at time \( t \). They adopted Monte Carlo simulation and least-squares to approximate the expected system values \( E_t[F_{t+1}(v_{t+1}; X_{t+1})] \) that appears in Eq. (8). The solution algorithm adopted by Zhao et. al. 2002 may be viewed as an extension of the least-squares Monte Carlo (LSMC) method. The LSMC was proposed by Longstaff and Schwartz (2001).

### 2.4 Two Probabilistic Life-Cycle Maintenance Models for Deteriorating Civil Infrastructures (Noorthwijk & Frangopol 2004)

These models are based on condition-based and reliability-based maintenance optimisation approach used in preventive maintenance strategies. Noorthwijk and Frangopol (2004) argued that the essential maintenance work which was defended on safety grounds was easy to justify. But the importance of work done for preventive maintenance strategies (both proactive and reactive) were more difficult to defend. The proactive maintenance strategy means applying maintenance before any indication of deterioration is apparent, while the reactive strategy means applying maintenance only after some deterioration is evident. They argued that using reliability-based approach would enable engineers to defend the importance of the preventive maintenance on a reliability basis. In the models, they compared the cost of preventive maintenance over essential (or collective) one. Then they balanced the cost of preventive maintenance against the cost of collective maintenance.

**Condition-based maintenance model**

They modelled the maintenance of structures as a (discrete) renewal process, whereby the renewals are the maintenance actions that bring a component back into its original condition or 'as good as new state'. After each renewal they started, in
statistical sense, all over again. A discrete renewal process \( \{N(n), n = 1, 2, 3, \ldots \} \) is a non-negative integer-valued stochastic process that registers the successive renewals in the time interval \((0,n]\). Let the renewal times \( T_1, T_2, \ldots \) be non-negative, independent, identically distributed random quantities having the discrete probability function \( Pr(T_k=i) = p_i, i=1,2,3, \ldots \), where \( p_i \) represents the probability of a renewal in unit time \( i \). The cost associated with a renewal in unit time is denoted by \( c_i, i = 1,2,3,\ldots \). The expected discounted cost over a bounded time horizon can be obtained with a recursive formula. To obtain this equation, Noortwijk & Frangopol (2004) conditioned on the values of the first renewal time \( T_1 \) and applied the law of total probability. The cost associated with occurrence of \( T_1 = i \) is \( c_i \) plus the additional expected discounted cost during the interval \((i,n]\), \( i = 1, \ldots, n \). Hence, the expected discounted cost over the bounded horizon \((0,n]\), denoted by \( E(K(n-i, \alpha)) \), can be written as

\[
E(K(n,\alpha)) = \sum_{i=1}^{n} \alpha^i p_i [c_i + E(K(n-1,\alpha))] \tag{9}
\]

\( \alpha = [1 + r/100]^{-1} \)

\( n = 1,2,3, \ldots, K(0,\alpha) = 0 \)

\( r = \text{the discount rate per unit time} \)

The expected discounted cost over an unbound horizon using discrete time renewal theory can be written as;

\[
\lim_{n \to \infty} E(K(n,\alpha)) = \frac{\sum_{i=1}^{\infty} \alpha^i c_i p_i}{1 - \sum_{i=1}^{\infty} \alpha^i p_i} \tag{10}
\]

For cost-optimal investment decisions, an optimal balance between the initial cost of investment and the future cost of maintenance, being the area of life-cycle costing, is of interest. In this situation, the monetary losses over an unbounded horizon are the sum of the initial investment \( c_0 \) and the expected unbound discounted future cost.

Noortwijk & Frangopol (2004) modelled the deterioration of an infrastructure as a stochastic process with gamma distribution, given as:

\[
Ga(x|\alpha,b) = \left[ b^\alpha / \Gamma(\alpha) \right] x^{\alpha-1} \exp\left\{ -bx \right\} 1_{(0,\infty)}(x) \tag{11}
\]

\( \Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt \) for \( \alpha > 0 \)

Where \( I_A = 1 \) for \( x \in A \) and \( I_A = 0 \) for \( x \not\in A \)

And the probability density function (PDF) of the amount of deterioration at time \( t \), \( X(t), t>0 \) is given by;

\[
f_{\dot{x}(t)}(x) = Ga(x|a^b, 0,1/\theta) \tag{12}
\]
The mean and variance of $X(t)$ are given by:

$$E(X(t)) = at^b, \quad Var(X(t)) = \theta at^b$$

For $\theta > 0$, $x > 0$ and $a > 0$.

A lifetime distribution, $F(t)$, of an infrastructure can be obtained from gamma distribution as given below, where shape parameter $\nu > 0$ and scale parameter $u > 0$.

$$F(t) = \Pr \{T \leq t\} = \Pr \{X(t) \geq y\} = \int_{x=y}^{\infty} f_{X(t)}(x) \, dx = \frac{\Gamma\left(\frac{at^b}{\theta}, y/\theta\right)}{\Gamma\left(\frac{at^b}{\theta}\right)}$$

(13)

Reliability-based maintenance model

In the reliability-based maintenance model, the term “reliability-based” means that the performance is quantified in terms of the reliability index or beta index $\beta = -\Phi^{-1}(p)$, where $p$ is the failure probability. Noorthwijk & Frangopol 2004 used eight random variables to represent the variability in input variables. These eight random variables include the initial reliability index $B_0$, the time of the damage initiation $T_i$, the reliability index deterioration rate $A$, the time of first application of preventive maintenance $T_{Pi}$, the time of reapplication of preventive maintenance $T_P$, the during of preventive maintenance effect on reliability $T_{PD}$, the deterioration rate of reliability index during preventive maintenance effect $\Theta$, and the improvement in reliability index (if any) immediately after the application of preventive maintenance $\Gamma$.

Noorthwijk and Frangopol (2004) used Monte Carlo simulation to generate random numbers for the probability distributions of the eight random variables to capture the propagation of uncertainties during the entire service of existing deteriorating structures.

The difference between the condition-based and reliability-based maintenance models is that the condition-based model used the condition profile defined by the initial condition index $C_0$, the time of condition deterioration initiation $T_{C}$, and the condition deterioration rate $\alpha_C$. While the reliability-based model used the reliability index profile defined by the initial reliability index $B_0$, the time of reliability index deterioration initiation $t_i$, and the reliability index deterioration rate $\alpha$.

2.5 Risk-Based Expenditure Allocation for Infrastructure Improvement (Ayyub & Popescu 2003)

Ayyub and Popescu 2003 developed risk-based expenditure allocation based on the probability-based reliability assessment and an analytical hierarchy method for multi-criteria.

They adopted the advanced second-moment reliability assessment method in which this method is capable of dealing with nonlinear performance functions and non-normal probability distributions (Ang & Tang 1975).
The analytical hierarchy process is used to derive ratio scales from both discrete and continuous paired comparisons. These comparisons may be taken from actual measurements or from a fundamental scale that reflects the relative strength of preference and feelings. The analytical hierarchy process is a nonlinear framework for carrying out both deductive and inductive thinking by taking several factors into consideration simultaneously and allowing for dependence and for feedback, and making numerical tradeoffs to arrive at a conclusion. In using the analytical hierarchy process to model a problem, one needs a hierarchy or network structure to represent that problem, as well as pair-wise comparisons lead to dominance matrices.

3. Proposed Methodology for Assessing Risk of Errors in Budget Estimates for Road Maintenance and Rehabilitation

It is evident from the literature review that currently many researchers suggested risk-based assessment methodologies and assumed the variability and probability distributions of budget/cost estimates for case studies. The degree of risk of errors in budget estimates has not yet reported in the literature.

This section presents a proposed method for assessing the degree of risk of errors in budget estimates for road maintenance and rehabilitation. The proposed method incorporates the variability of road asset conditions and other critical input parameters of the road network in the assessment.

The first step in this method is to define a performance function which transforms input variables into maintenance and rehabilitation budgets. The second step is to define the input variables. A performance function for a discount budget estimate for road maintenance and rehabilitation for a life-cycle budget/cost estimate of a road network can be written as:

\[ G = \frac{1}{(1 + r)^t} \sum_{i=1}^{m} f(Z_1 Y_{1,i}, Z_2 Y_{2,i}, \ldots, Z_m Y_{m,i}) \]  

(14)

Where \( G \) is the total budget expressed in terms of probability distribution. \( m \) is the number of critical input variables, the variability of which is considered in the analysis. \( Y_{1,n,t}, Y_{2,n,t}, \ldots, Y_{m,n,t} \) are random variables of input variables with known probabilities of section \( n \) in year \( t \). \( Z_1, Z_2, \ldots, Z_m \) are random transform functions representing model errors in prediction. \( n \) is the number of road sections used in the analysis. \( t \) is the total year used for the life-cycle budget estimates. \( r \) is the discount rate.

The calculation of Equation 14 is the subject of determining the relationship between input statistics and output statistics (i.e. how the variability of input variables affects the variability of output variables). The calculation of the probability of Equation 14 becomes difficult since the transform function is highly complicated. It involves establishing deterioration prediction models of road conditions; quantifying road usage and forecasting the incremental road usage into the future; and optimising different budget scenarios to obtain optimal budget estimates.

To this end, a simulation method is desirable for the statistical assessment of the input and output relationship. A simplified sampling technique such as the Monte Carlo Simulation technique (Gary an Travers, 1987) may require a larger number of
data to be sampled to represent an overall variability of an input variable. The Latin hypercube sampling technique, as extensively studied by Iman and Conover (1980), provides a satisfactory method for selecting small samples of input variables so that good estimates of the means, standard deviations and probability distribution functions of the output variables can be obtained.

In the proposed study, Highway Development and Management (HDM-4) System software will be employed as the calculation tool to determine the relationship between input and output variables and to calculate output statistics.

HDM-4, developed by the International Study of Highway Development and Management (ISOHDM), is a globally accepted pavement management system. HDM-4 is a computer software package used for planning, budgeting, monitoring and management of road systems. There are three analysis options in HDM-4. These analysis options include (1) Strategy Analysis, (2) Program Analysis and (3) Project Analysis. The Strategy Analysis Option was employed in this study in assessing the effects of the variability of pavement strength on the estimate of maintenance and rehabilitation budgets.

**Framework for Risk of Errors Assessment for Budget Estimates**

Figure 1 shows the schematic chart of the framework for risk of errors assessment (Piyatrapoomi & Kumar 2003), and can be summarised as follows;

1. The first task is to identify critical input variables that significantly affect the budget estimates. In this study, HDM-4 software will be used for the analysis.
2. Establish probability distributions and statistical information (means, standard deviation and etc.) of critical input variables.
3. Divide the road network into small sections and assign the probability distributions of the critical input variables for each road section.
4. Sample observation values of input variables of each road section. In this study, the Latin Hypercube Sampling Technique will be employed.
5. Use HDM-4 models to predict road deterioration for a life-cycle analysis. Use the input probability distributions of the critical variables and deterioration prediction models to predict road conditions in future, and estimate output statistics of budgets for maintenance and rehabilitation for a life-cycle budget estimate. In this study, Highway Development Management System (HDM-4) will be employed for statistical analyses to obtain the output statistics.
6. Establish the probability distribution and determine the statistical information of output parameters.
7. Incorporate predicting model errors into the probability distribution of the output parameters.
8. Assess the probability of the interest or risk of errors in the budget estimates from the outcome probability distribution and statistical information.
1. Identify critical input variables.

2. Establish probability distributions of critical input variables for the road network to be analysed.

3. Segment of the road network and assign the probability distributions of the critical input variables for each road section.

4. Sample critical input variable values from the probability distributions for each road section. Latin hypercube sampling will be used.

5. Use the input probability distributions of the critical variables and deterioration prediction models to predict road conditions in future, and estimate output statistics of budgets for maintenance and rehabilitation. HDM-4 will be used for this analysis.
4. Conclusions

A review of the literature on the methodologies for assessing risk of errors in life-cycle budget estimates for infrastructure asset management was conducted. It was found that risk-based budget/cost estimates had become an important issue. Most researchers made assumptions about the variability and probability distributions of budget/cost estimates. A methodology for assessing risk of errors in life-cycle cost budget estimates for road maintenance and rehabilitation was formulated in this study based on the review of the literature. This methodology aims at identifying the degree of errors in budget/cost estimates due to the variability of critical input variables. The proposed method incorporates the variability of critical road asset conditions along the road network into the analysis. The outcome of the analysis will be the probability distribution of a life-cycle budget estimate in which the overall critical variability of input variables is built into or factored into the analysis. From the output probability distribution of budget estimates, budget analysts will be able to refine the accuracy of budget estimates. For instance, a budget could be selected for a 90% or 95% confidence that it will be not exceeded, or a lower budget could be selected with a corresponding higher degree of risk of being exceeded. The practical contribution of this study is that budget analysts will be aware of the degree of risk of errors when a budget is selected.
List of References


Author Biography

Noppadol Piyatrapoomi obtained his Ph.D. degree from the University of Melbourne. He has practiced as a civil and structural design engineer for ten years before he joined RMIT University Melbourne Australia in 2002. His research interests include the application of risk and reliability in decision-making for infrastructure asset management; assessment of public risk perception on engineering investments; risk and reliability assessment of structures; seismic risk and reliability assessment of structures; the application of an evolutionary method for data analysis. He developed an evolutionary method of data analysis during his Ph.D. study. This method can be used to refine existing functions and develop new formulas by using probability, statistical and risk assessment theories in the analysis. The method provides a more fine-grained analysis and yields more accurate results and better fitness of data than commonly used methods, such as regression or correlation analyses.